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Mathematical modelling of wind-sand multiphase flows

Luigi Preziosi[1,2]

[1]*Department of Mathematical Sciences, Politecnico di Torino, Torino, Italy,* *luigi.preziosi@polito.it*
[2]*Windblown Sand Modelling and Mitigation joint research group, Italy,* *wsmmm@polito.it*

SUMMARY:
The transport of particulate by wind constitutes a relevant phenomenon in environmental sciences and civil engineering, because erosion, transport and deposition of particulates can cause serious problems to human infrastructures. From a mathematical point of view, modeling procedure for this phenomenon requires handling the interaction between different constituents, the transfer of a constituent from the air to the ground and viceversa, and consequently the ground-surface interaction and evolution. Four phenomena then contribute to wind-induced sand movement and in particular to the formation and evolution of dunes and of sand accumulation: erosion from the sand bed, transport by the wind, sedimentation due to gravity, and sand grain slides occurring when the slope of the accumulated sand exceeds a critical repose angle. In the following we will describe a comprehensive mathematical structure than takes care of all these aspects leading to realistic simulations. In particular, we will discuss in more detail the model that is in charge of describing the sliding of accumulated sand particles.

Keywords: wind-blown particles, transport phenomena, sand avalanche

1. MODELLING FRAMEWORK

Referring to Figure 1, sand entrainment in the wind is due to the lift-off of sand grains from the ground. The smallest particles remain in suspension for a longer period slowly sedimenting, while bigger particles, whose aerodynamic behavior is also strongly affected by their shape, follow ballistic trajectories before impacting again on the ground, transferring momentum to other particles which are then ejected from the soil. In this way saltating particles are also able to displace particles that are too heavy to be lift-off by the action of the wind, and induce very short trajectories that trigger a transport phenomenon called reptation. Another process involving big particles is creep, that consists in particle rolling and sliding on the surface made of other deposited particles. Among all transport mechanisms, saltation involves about 80% of the total mass (Kok et al., 2012). The presence of sand grain suspended in the air makes the wind flow multiphase with the solid phase that is sedimenting while it is transported. Both erosion and sedimentation determine the evolution of the ground surface, which makes the mathematical problem a free-boundary value problem. As sand accumulates, the particles on the surface may slide down, representing a phenomenon that strongly contributes to the shaping of the ground surface. In the following, referring to Lo Giudice, Nuca, et al., 2019 for a review, we will briefly describe in a modular way a mathematical framework that can handle all these aspects leading to realistic simulations.

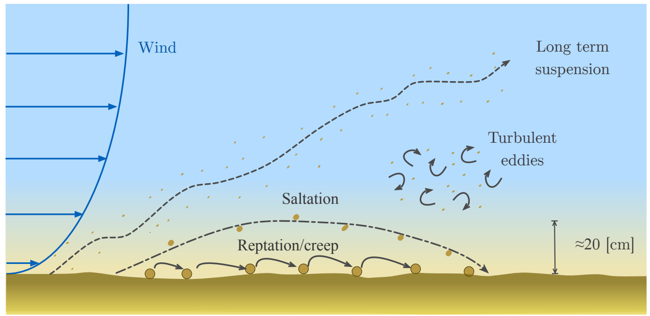

Figure 1. Schematic representation of different wind-driven transport modes.

1.1. MULTIPHASE FLUID-DYNAMICS

Sandy-wind is modeled as a multiphase-system constituted by the air (the carrying phase) and sand (the dispersed phase). Considering that the flow is turbulent, we model the air-phase using Unsteady Reynolds Averaged Navier-Stokes equations coupled with the Shear Stress Transport (SST) formulation of k-ω turbulence model to close the system of equations. For sake of conciseness we will not report it here. However, we mention that the Reynolds Averaged approach with the selected turbulence model has already been used for dune aerodynamics analysis in Bruno and Fransos, 2015; Liu et al., 2011, and for bluff-body aerodynamics in deserts in Bruno, Fransos, and Lo Giudice, 2018. The dispersed phase is considered as a passive scalar modelled by the conservation equation for sand volume fraction ϕ_s

$$\frac{\partial \phi_s}{\partial t} + \nabla \cdot \left[\alpha \phi_s \bar{\mathbf{u}}_f + \phi_s \mathbf{u}_{sed} - \nu_{eff} \phi_s^{k-1} \nabla \phi_s \right] = 0 \qquad \text{with} \quad k \geq 1 \,. \tag{1}$$

So, the flux is the combination of advection by wind, sedimentation effects due to gravity, and diffusive flux due to random collisions. It can be noticed that, following the discussion in Balachandar and Eaton, 2010, the advection term is taken to be proportional to the average wind velocity $\bar{\mathbf{u}}_f$, according to experimental data. As discussed in Preziosi et al., 2015, the sedimentation velocity $\mathbf{u}_{sed}$ can be obtained by plotting the experimental relationship between particle Reynolds number and drag given for instance in Barnea and Mizrahi, 1973 in terms of a relationship between the grain size and the particle Reynolds number $Re_p = \frac{u_{sed} d}{\nu_f}$, where d is the particle diameter. Finally, through the effective viscosity ν_{eff} the last term takes into account of the mixing-diffusive contribution due to the air viscosity ν_f, to the turbulent viscosity ν_t and to the random collisional-interactions ν_s. ν_{eff}/u_{sed} determines the order of magnitude of the saltation layer height.

1.2. SALTATION INFLUX

As already mentioned the crucial mechanism of sand transport in the air is the lift-off of sand grain off the ground. This triggering transport mechanism, named saltation, is a phenomenon that takes place if the wall shear stress τ on the ground surface generated by the blowing wind exceeds a threshold value τ_t. For instance, the vertical flux expression discussed in Ho et al., 2013 based on experiments on saltating grains writes as

$$-\nu_{eff} \phi_s^{k-1} \nabla \phi_s \cdot \mathbf{n} = A_H \hat{\rho}_f \sqrt{\frac{d}{g}} (u^{*^2} - u_t^{*^2})_+ \,, \tag{2}$$

where A_H depends on physical properties of the sand and $(f)_+ := (f+|f|)/2$ stands for the positive part of f. In this expression that acts as a boundary condition for (1), as usually done in the literature, instead of the wall shear stress τ the shear velocity u^* defined as $u^* = (|\tau|/\hat{\rho}_f)^{0.5}$ is used. We notice that the threshold value strongly depends on particles properties such as size and on chemical interactions among them, in conjunction with environmental conditions such as humidity, size, and wetting. In Lo Giudice and Preziosi, 2020 a modification taking care of the local slope of the ground surface is suggested.

1.3. SAND SLIDING

The morphodynamic evolution of the shape of dunes and piles of granular material is largely dictated by avalanching phenomena, acting when the local slope gets steeper than a critical repose angle. In Lo Giudice, Giammanco, et al., 2019 the following parabolic equation is proposed

$$\frac{\partial h}{\partial t} = \nu \nabla \cdot \left[f_{sl}(|\nabla h|) \frac{\nabla h}{|\nabla h|} \right] + q \,, \tag{3}$$

where ν is an effective diffusion coefficient and q takes into accout of external volume supplies of sand per unit area. The mathematical model is based on a reduction of a mass balance equation obtained assuming that the thickness of the sliding layer is small and that the grains move along the direction of steepest descent. The existence of a repose angle triggering the motion of the sand grains reflects into the fact that, as we shall soon see, the sliding term f_{sl} vanish for slopes, and therefore $|\nabla h|$, below a threshold value. This gives to the parabolic equation (3) a degenerate character. In fact, if the speed is given by the limit velocity of a body sliding down a slope under the action of gravity, Coulomb friction, and a drag force taken to be proportional to the sliding speed the sliding term f_{sl} can be taken as

$$f_{sl}(|\nabla h|) = \frac{(\tan\alpha - \tan\alpha_{cr})_+}{\sqrt{1+|\nabla h|^2}} \qquad \text{where} \quad \tan\alpha = |\nabla h| \,. \tag{4}$$

We notice that, due to the presence of the positive part g_+, f_{sl} vanishes when $|\nabla h| \leq \tan\alpha_{cr}$, where α_{cr} is the repose angle. In alternative, according to the viscoplastic constitutive equation used to describe sand behaviour, following Nuca et al., 2021 one has

$$f_{sl}(|\nabla h|) = \frac{\sin\alpha_{cr}}{\sin\alpha}\left(2 + \frac{\sin\alpha_{cr}}{\sin\alpha}\right)\left[\left(\frac{\sin\alpha}{\sin\alpha_{cr}} - 1\right)_+\right]^2, \tag{5}$$

for an Herschley-Bulkley model (the Bingham case being recovered when $\gamma = 1$) and

$$f_{sl}(|\nabla h|) = \frac{\sin^2\alpha_{cr}}{\sin^2\alpha}\left(10\frac{\sin^{3/2}\alpha}{\sin^{3/2}\alpha_{cr}} + 6\frac{\sin\alpha}{\sin\alpha_{cr}} + 3\frac{\sin^{1/2}\alpha}{\sin^{1/2}\alpha_{cr}} + 1\right)\left[\left(\sqrt{\frac{\sin\alpha}{\sin\alpha_{cr}}} - 1\right)_+\right]^3, \tag{6}$$

for a Casson model. The model results particularly simple to implement and is very versatile, being able to naturally describe to the movement of sand in presence of walls, open ends, columns, doors, and in complicated geometries. As an example, in Figure 2a we show the results obtained using Coulomb closure (4) when simulating the sand entering a room through a door as shown in Figure 2b. From the structure of the equation it is clear that in absence of any mass input q, every

Figure 2. Sand pile entering a room: simulation on the left and results on the right.

subcritical configuration with $h(\mathbf{x})$ such that $|\nabla h| \leq \tan\alpha_{cr}$, $\forall \mathbf{x}$ is a stationary solution, because f_{sl} always vanishes being the angle of steepest descent $\alpha < \alpha_{cr}$. On the other hand, starting from an initial condition such that $|\nabla h| > \tan\alpha_{cr}$ everywhere, then the evolution will tend to a solution of $f_{deg}(|\nabla h|) = 0$. Now, regardless of the closure assumption used, and therefore of the sliding model, this configuration is a solution of the eikonal equation $|\nabla h| = \tan\alpha_{cr}$, which then represents a robust feature of all the constitutive closures proposed above.

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REFERENCES

Balachandar, S. and Eaton, J., 2010. Turbulent dispersed multiphase flow. Annual Rview of Fluid Mechanics 42, 111–133.

Barnea, E. and Mizrahi, J., 1973. A generalized approach to the fluid dynamics of particulate systems: Part 1. General correlation for fluidization and sedimentation in solid multiparticle systems. Chemical Engineering Journal 5, 171–189.

Bruno, L. and Fransos, D., 2015. Sand transverse dune aerodynamics: 3D coherent flow structures from a computational study. Journal of Wind Engineering and Industrial Aerodynamics 147, 291–301.

Bruno, L., Fransos, D., and Lo Giudice, A., 2018. Solid barriers for windblown sand mitigation: Aerodynamic behavior and conceptual design guidelines. Journal of Wind Engineering and Industrial Aerodynamics 173, 79–90.

Ho, T., EI Moctar, A., and Dupont, P., 2013. Scaling laws in aeolian sand transport. Physical Review Letters 106, 094501.

Kok, J., Parteli, E., Michaels, T., and Karam, D., 2012. The physics of wind-blown sand and dust. Report Progress in Physics 75, 106901.

Liu, B., Qu, J., Zhang, W., and Qian, G., 2011. Two-equation eddy-viscosity turbulence models for engineering applications. Journal of Wind Engineering and Industrial Aerodynamics 99, 879–888.

Lo Giudice, A., Giammanco, G., Fransos, D., and Preziosi, L., 2019. Modelling sand slides by a mechanics-based degenerate parabolic equation. Mathematics and Mechanics of Solids 24, 2558–2575.

Lo Giudice, A., Nuca, R., Preziosi, L., and Coste, N., 2019. Wind-blown particulate transport: A review of computational fluid dynamics models. Mathematics in Engineering 1, 508–547.

Lo Giudice, A. and Preziosi, L., 2020. A fully Eulerian multiphase model of windblown sand coupled with morphodynamic evolution: Erosion, transport, deposit and avalanching. Applied Mathematical Modeling 79, 68–84.

Nuca, R., Lo Giudice, A., and Preziosi, L., 2021. Degenerate parabolic models for sand slides. Applied Mathematical Modeling 89, 1627–1639.

Preziosi, L., Fransos, D., and Bruno, L., 2015. A multiphase first order model for non-equilibrium sand erosion, transport and sedimentation. Applied Mathematics Letters 45, 69–75.